Simulation of Catastrophic Failure in a Residual Stress Field

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(Received 8 January 2021; revised 20 May 2021; accepted 17 June 2021; published 3 August 2021)

Residual stress has been empirically utilized for industrial applications to control material strength and shape of fragments. The interaction between the dynamically growing cracks and the residual stress field is sufficiently complicated to prevent us from building effective models. To rigorously evaluate the release and redistribution of residual stress in the dynamic fracture process, we develop a mathematical model and a numerical analysis method for the dynamic fracture in a residual stress field. Our methodology is simple and rigorous and applicable regardless of materials and scales.

DOI: 10.1103/PhysRevLett.127.064301

Control of the residual stress field is a significant problem in industrial applications because the residual stress field is highly related to the strength of the bulk materials. The tensile residual stress produces a stress concentration at crack tips and promotes failure. Especially under high tensile residual stress, the crack rapidly propagates and causes catastrophic fragmentation of materials [1,2]. However, the intentional introduction of surface compressive residual stress by surface machining or a finishing process [3,4] prevents crack initiation and growth. So far, because the understanding of this fracture behavior in a residual stress field is only intuitive, the controlled residual stress field is empirically employed in manufacturing techniques to improve material strength.

Fracture behavior in a residual stress field has also attracted scientific attention. However, dynamic fracture in a residual stress field brings substantial theoretical complexity because we have to solve the mutual interaction among crack propagation, change in the residual stress field, and generation of the elastic wave [5]. Moreover, although the systems and devices for the full-field measurement of the stress field have been developed in recent years [e.g., digital image correlation (DIC) [6–8] and high-speed digital photoelasticity [9–13]], these experimental approaches are limited to the evaluation of the outer surface residual stress field (DIC) or the residual stress intensity averaged over the thickness of the specimen (photoelasticity).

In view of this situation, attempts have been made toward numerical analysis of crack growth in various materials with a residual stress field [14–17]. In spite of these attempts, the achievements of previous work are mainly confined to the evaluation of the quasistatic propagation of a single crack. The dynamic propagation of multiple cracks in a residual stress field is still unsolved and highly challenging.

Mathematical model.—We first develop the mathematical model and numerical analysis method for the dynamic fracture in a residual stress field by applying the discretization scheme proposed in the particle discretization scheme finite element method (PDS-FEM) to the solid continuum with a residual stress field [18–20]. We assume that the elastic deformation is the only source of the residual stress in the solid material. The total strain tensor ϵ^t_{ij}, which represents the total deformation from the initial stress-free state, and the residual stress tensor σ_{ij} are related by $\sigma_{ij} = c_{ijkl}(\epsilon^t_{kl} - \epsilon^p_{kl})$, where c_{ijkl} is the elasticity tensor and ϵ^p_{kl} is the permanent inelastic strain tensor. In this Letter, all the strains that do not contribute to the generation of the elastic stress (i.e., residual stress) in the linear elastic material are referred to as the permanent inelastic strain ϵ^p_{kl}.

Let us consider a deformation problem for the homogeneous isotropic linearly elastic body Ω with the prescribed distribution of the inelastic strain. In PDS-FEM, the analysis domain is discretized by using a pair of conjugate geometries corresponding to a set of nodes $\{x^\alpha\}$: Voronoi tessellations and Delaunay tessellations. Here, the superscripts α and β respectively represent the variables for the αth Voronoi tessellation and the βth Delaunay tessellation. The discretized strain energy $\hat{J}$ stored in Ω is

$$\hat{J} = \sum_{\beta=1}^{M} \frac{1}{2}(\epsilon^{t\beta}_{ij} - \epsilon^{p\beta}_{ij}) c_{ijkl} (\epsilon^{t\beta}_{kl} - \epsilon^{p\beta}_{kl}) \Psi^\beta, \quad (1)$$

where M is the number of Delaunay tessellations and Ψ^β is the volume of the βth Delaunay tessellation. The summation convention is employed for subscripts throughout this Letter. This discretized strain energy $\hat{J}$ is expressed in terms of the total displacement $u^{t\alpha}_i$ by introducing the displacement-strain relationship in

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